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A Survey of the Literature on Production Scheduling as it Pertains to Flexible Manufacturing Systems

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FOREWORD

The National Bureau of Standards (NBS) Center for Manufacturing Engineering (CME) has established an Automated Manufacturing Research Facility (AMRF) to support research and experimentation in automated manufacturing and computer interface standards for the computer integrated factory of the future. This research facility provides a basic array of manufacturing equipment and systems configured into a small batch manufacturing system that can be used by researchers from NBS, industry, academe, and other government agencies in these efforts to study and improve automated factories.

The Center for Applied Mathematics (CAM) is engaged in a long-term research effort with the Center for Manufacturing Engineering to design, develop, and test scheduling and routing algorithms for use in the AMRF.

The AMRF represents a unique and exciting opportunity in the history of the development of solution techniques for scheduling and routing problems. At present, optimal solutions to these problems can be found only in cases where there are a small number of machines and a small number of parts to be scheduled for production. Large problems are rarely solved to optimality because of the combinatorial increase in their computational complexity. Moreover, small problems solved by currently available computer programs typically require more time than the demands of a real-time manufacturing system will allow.

The approach traditionally used to create real-time schedules avoids the issue of optimality and obtains "acceptable" solutions by heuristic methods. There is a large body of literature on scheduling and routing heuristics and it increases regularly. However, few of these heuristics have been thoroughly tested in practice, because of the expense to experiment with in-place manufacturing facilities. Until recently, no test environment real enough to produce useful answers was available.

The AMRF is such an environment. Its operating system, called the AMRF Hierarchical Control System Emulator (HCSE), can configure virtual machines, cells and shops, and thus can be used as a flexible test bed for testing heuristic solutions to real-time scheduling and routing problems for these facilities. The HCSE can provide computer programs to replace any element in the AMRF, from controlling computers to individual machines or robot carts. To the rest of the system, the emulation apppears exactly the same as what it replaces, so that risky changes to the system can be tested without endangering expensive equipment. This means that we have at NBS a unique national research capability which can and will be used to try out new ideas in many areas of automated manufacturing.

This report is the first in a series of papers planned to document efforts to solve scheduling and routing problems in the AMRF. It is a thorough review of the existing literature on machine scheduling as it relates to flexible manufacturing systems, and is the logical place to begin such a series.

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A SURVEY OF THE LITERATURE

ON PRODUCTION SCHEDULING

AS IT PERTAINS TO

FLEXIBLE MANUFACTURING SYSTEMS

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A Survey Of The Literature On Production Scheduling As It Pertains To Flexible Manufacturing Systems

ABSTRACT

This paper presents a survey of the existing literature on machine scheduling from the perspective of its potential applicability to the environment of flexible manufacturing. It is the first of a series of papers planned to document research on the development of a scheduling system for the Automated Manufacturing Research Facility (AMRF) at the National Bureau of Standards.

An overview of the hierarchical production planning process is given. Scheduling appears at the lowest level of this hierarchy in the sense that it is carried out at the lowest levels of the organizational hierarchy where current information is available and execution is carried out repetitively within relatively short time horizons.

The scheduling problem is classified along the dimensions of order characteristics, processing environment, and scheduling objective. The paper covers deterministic, non-preemptive scheduling of open shops, since a typical flexible manufacturing system (FMS) operates under these conditions. Both due-date based and flow-time based objectives are addressed for the processing environments of single stage single machine, single stage parallel machines, flow shops and job shops. The research done in the area of assembly line balancing is similarly covered since it is possible to treat an FMS as a transfer line for repetitive discrete manufacture.

The FMS-related management problems are classified into, i) Design problems, ii) Planning problems, and iii) Scheduling and Control problems. The analytical approaches to these problems have focussed primarily on the objectives of maximizing production rate, minimizing in-process inventory, and maximizing machine utilization.

With the exception of a few simple cases, the scheduling problems in both conventional and flexible manufacturing environments belong to a set of 'hard' combinatorial problems.

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1 INTRODUCTION

1.2 Purpose of the Review

This paper is the first in the series of reports on the scheduling of the Automated Manufacturing Research Facility (AMRF) at the National Bureau of Standards. The paper surveys the existing literature on machine scheduling from the perspective of its applicability to flexible manufacturing systems (FMS). Since research efforts in the past addressed primarily the issues arising in conventional manufacturing, this paper attempts to identify and review those aspects of the scheduling literature which are potentially relevant to the environment of flexible manufacturing.

We begin with a brief overview of the production planning process. After reading this overview, the reader should understand that the focus of the current research effort is to develop real-time scheduling methods which appear conceptually at the lowest level of the described hierarchical production planning process.

1.2 The Production Planning Process

Production can be defined as the process of converting raw materials into finished goods. The major inputs into the production process, aside from raw materials, are human resources, facilities, such as plant and equipment, and information regarding the nature of the conversion process. Buffa and Miller (1979) classify production systems along the dimensions of the order characteristics and the nature of the processing environment. Order characteristics refer to the manner in which the production orders are generated. In an *open shop*, the job orders are based on actual customer requirements. There is no attempt made to produce any item for replenishment of inventory: In contrast, in a *closed shop*, job orders are oriented towards stock replenishment and production quantities are

decoupled from actual sales through the use of the finished goods inventory.

The processing environment could either be one of continuous-manufacture, which are also referred to as discrete-repetitive manufacturing systems, or one of intermittent-manufacture. Continuous-manufacture systems are geared to produce a continuous stream of products. These systems are suitable for manufacturing a low variety of high-volume products. Assembly lines in automobile industries are typical examples of continuous-manufacture systems. In the scheduling literature, these systems are referred to as flow shops. Intermittent-manufacture systems, also called job shops, are designed for low-volume, batch production. Their efficacy lies in their ability to cater to a large variety of products which through the use of general purpose equipment, can be readily changed from the production of one product to another, and a flexible workforce.

Flexible manufacturing systems attempt to combine the throughput capability of flow shops with the flexibility of job shops. The versatility of computer-controlled machine tools and the relatively short changeover times provide FMS's theoretically with the capability to manufacture efficiently a large variety of parts in small to medium batch sizes without affecting productivity seriously. Figure 1 depicts the economically ideal relative position of flexible manufacturing systems with respect to flow shops and job shops. However, given their characteristics, it is possible to operate an FMS in the extreme regions as a pure job shop or as a dedicated flow shop. As a result of the wide range of possible operating modes for an FMS, the literature survey contained herein covers the full range of scheduling, from scheduling job shops to scheduling flow shops.

Regardless which production system is being utilized, it is necessary to marshal the human, technological and material resources at the correct time and place to translate effectively production demands into real finished products. This management function of translating production forecasts or orders into workable plans, and the monitoring of these plans to ensure they are implemented, is generally referred to as production planning. Like any other management function, it requires making several decisions at

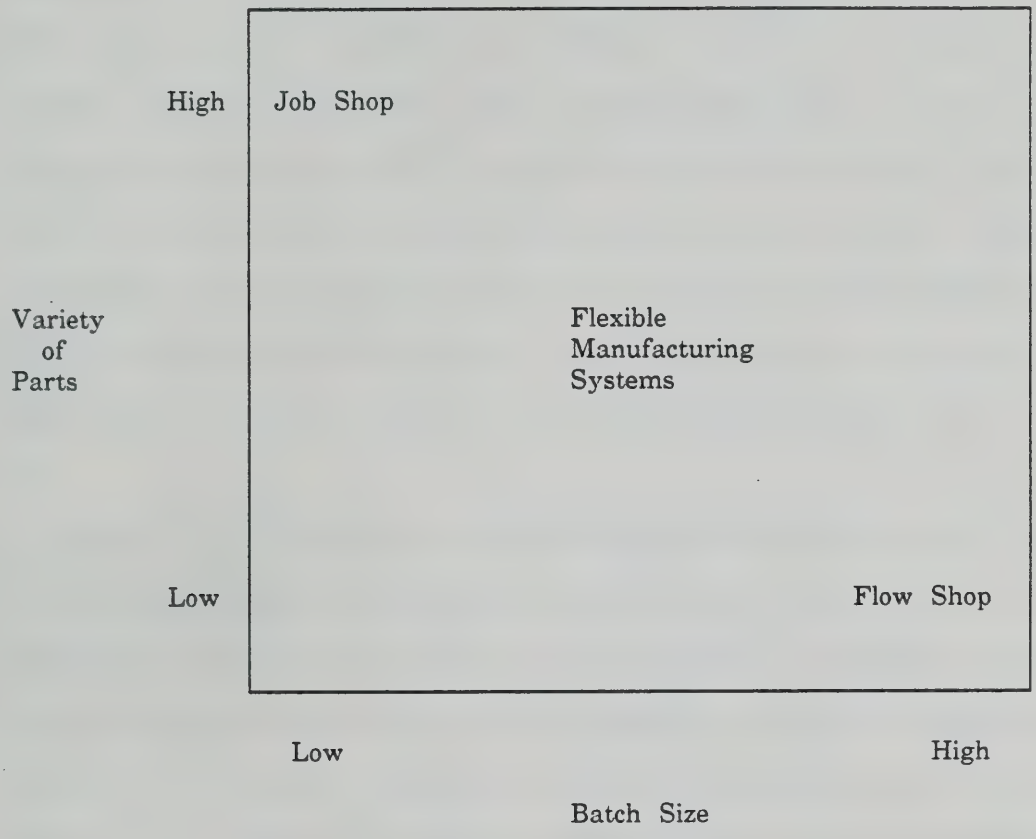
High
Job Shop
Variety
of
Parts
Flexible
Manufacturing
Systems
Low
Flow Shop
Low
High
Batch Size

Figure 1

different echelons of the organizational hierarchy, and across varying time-horizons.

Anthony and Dearden (1980) suggest a hierarchical approach to analyze the managerial decision making process. The highest level of *strategic planning* requires the formulation of long-term objectives, deciding upon the acquisition of resources to meet these objectives, and specifying the policies to administer the usage of these resources. These decisions usually span long time-horizons and are made at the top management level. In the context of production management, decisions at this level are concerned with the location of manufacturing and distribution facilities, specification of product quality and of inventory policies, and the assessment of the categories of human skills required. While a detailed discussion of all of these issues is beyond the scope of this paper, a brief review of the research done on facility location theory is presented in Section 1.3.

The next level of *management control* ensures that the resources are utilized effectively to meet organizational goals. Management control decisions are usually made at the middle management level and cover medium-range time-horizons. In a production system, this level is concerned with the formulation of the aggregate production plan to meet demand forecasts at minimum cost. Aggregate planning decisions relate to the specification of inventory levels, utilization of work force, machine utilization levels, and determination of production quantities during each period of the planning horizon. Aggregate measures of volumes and capacities such as generic units of output, tons per day, man-hours, are usually used at this level of analysis. Mathematical modeling approaches to the aggregate planning problems are discussed in Section 1.4.

The lowest level of *operational control* is concerned with the efficient execution of the tasks specified by the management control process. Decisions at this level span short time-horizons and are made at the lower-middle level of management. Scheduling forms a part of the operational control process in a production system. For a given aggregate plan, scheduling involves efficient allocation of machines to jobs, or operations within jobs, and

the subsequent time-phasing of these jobs on individual machines. Section 1.5 presents a discussion on the nature of the scheduling problems. A schematic diagram depicting the relationships among these three levels is shown in Figure 2.

In discrete-repetitive and medium size batch manufacturing, two intermediate steps, which fall between aggregate planning and detailed scheduling, are normally followed in the production planning process. The first is the development of the master production schedule (MPS) which is time-phased specification of the volume (units) required of each end-item (product) type. This MPS usually extends one to three months into the future and is based on forecasts of finished goods demand as well as firm orders. The MPS is usually updated monthly for one, two, and three months into the future, in what is referred to as a rolling production planning horizon. For each future month, production demands are normally broken down into weekly time intervals or "time-buckets".

In the second step, the MPS is translated into time-phased purchase and production orders by taking into account ordering and production lead times, inventory status, and product structure. The latter is specified by a bill of materials. The translation process today is called exploding the bill of materials into shop orders, and is frequently carried out by a computer-based information system called a Materials Requirement Planning (MRP) system. The major output from the MRP system is a set of shop orders that specify how many of each component need to be made or purchased, and in which time period (usually which week) orders need to be placed or manufacturing initiated.

Finally, these shop orders are processed either manually (by a production planning staff member or shop foreman), or by a computer-based shop-floor scheduling program into detailed schedules that specify fairly precisely when, how (machines, tooling, etc.), and by whom each job is to be made.

Readers interested in the general overviews of the production planning process and MRP systems are referrd to Chase and Aquilano (1985), and Meal (1984). A more technical approach, with a more extensive bibliography, can be found in Hax and Candea

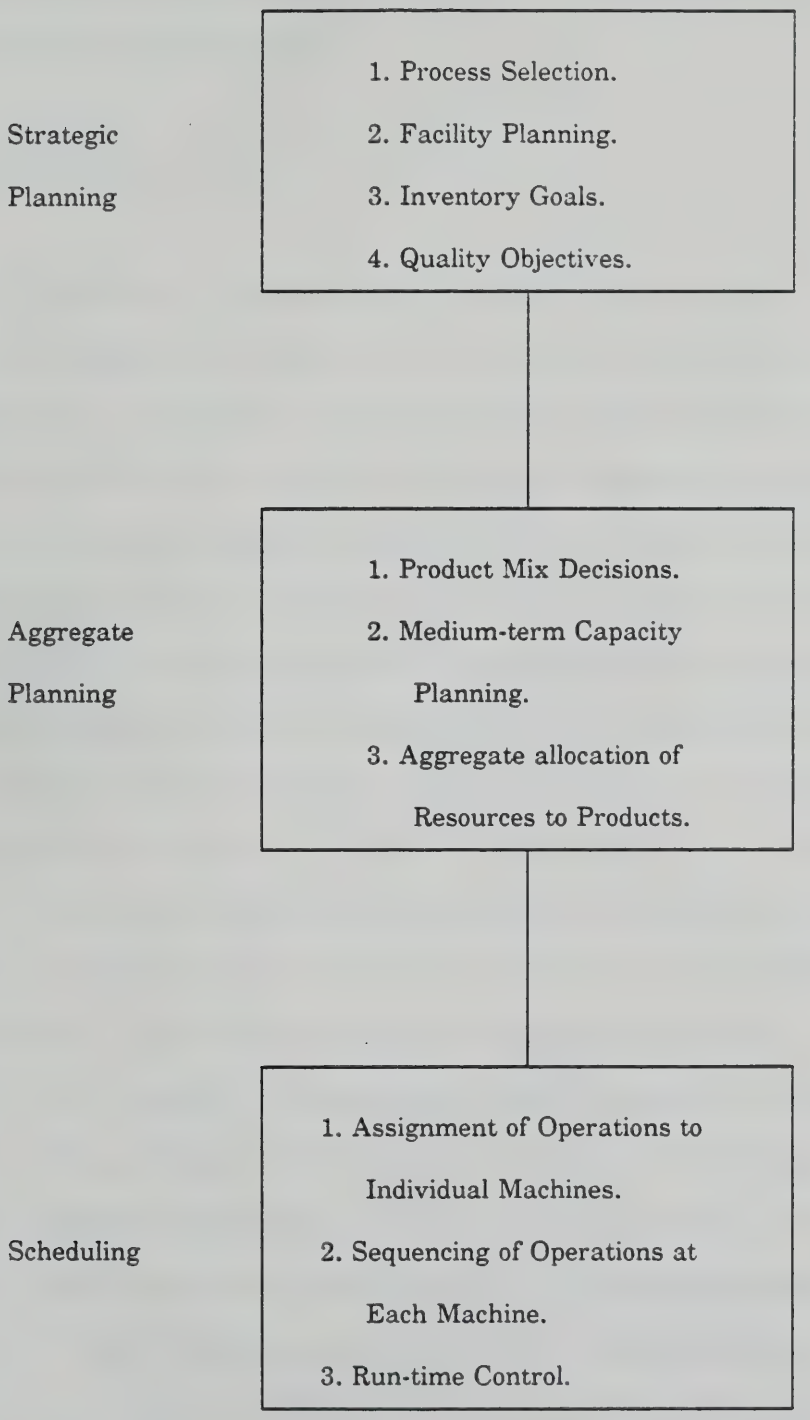
Strategic Planning
1. Process Selection.
2. Facility Planning.
3. Inventory Goals.
4. Quality Objectives.
Aggregate Planning
1. Product Mix Decisions.
2. Medium-term Capacity Planning.
3. Aggregate allocation of Resources to Products.
Scheduling
1. Assignment of Operations to Individual Machines.
2. Sequencing of Operations at Each Machine.
3. Run-time Control.

Figure 2

(1984). The remaining introductory sections in this paper will simply give an overview to the major topics of facility planning, aggregate planning, and scheduling, the latter being the primary area of interest for this research.

1.3 Facility Location

Planning for increases in manufacturing and distribution capacities requires consideration of the location of the additional capacities. Decisions on facility location can also be triggered by a geographic shift in the market and/or the raw material suppliers. While a new firm has to acquire new plants and warehouses, an ongoing firm has the option of on-site expansion as well. Schmenner (1979) has dealt with some of the strategic considerations involved in deciding between on-site expansion and new facility location. A new plant facilitates the introduction of new processes, improvement in plant layout and material handling systems, and control on the size of the work force. Also, with new plants, it is easier to manage new products and service geographic market growth. On the other hand, on-site expansion imposes less financial burden and generates economies of scale. Also, fewer managers are typically required to supervise the additional capacity.

Mathematical modeling approaches to the facility location problem have addressed the issue of minimizing the costs involved in locating new facilities with respect to a set of existing facilities, which includes the existing and potential customers and the raw material suppliers. Hax and Candea (1984) identified seven factors which determine the formulation of an appropriate model. These are: i) the nature of the possible sites – the continuous space approach assumes that the facility can be located anywhere on a two-dimensional space, while the discrete space approach considers only specific potential sites; ii) optimization criteria, which could either be the minimization of the average cost over all the facilities or the minimization of the maximum cost of locating any one facility; iii) the nature of the problem, which could be a pure location problem, a pure allocation problem or

a combination of both; iv) the number of facilities to be located; v) the number of products being manufactured and distributed, vi) the number of distribution stages. and, vii) the capacity restrictions on the facilities. In order to give the reader a sense of the research in this area, the first factor "nature of the possible sites" is explored in somewhat more detail in Sections 1.3.1 and 1.3.2. Voluminous literature exists on all of these seven factors, and the interested reader is referred to Hax and Candea (1984) and Francis and White (1974) for research results and bibliography.

1.3.1 Continuous Space Approach

In this approach, the cost of material movement is assumed to be dependent only upon the distance between the facilities. The rectilinear distance model assumes that the travel occurs along paths set in a rectangular pattern. For the single-facility location problem, the x coordinate (y coordinate) of the optimal location is the median location, i.e., the material movement to the left of (below) and to the right of (above) the new facility location is less than or equal to one half the total material movement. For locating multiple facilities, the optimal sites can be determined by solving a linear programming problem for each of the two coordinates.

In the squared Euclidean distance model, the transportation cost is assumed to be a quadratic function of the Euclidean distance. The optimal solution for the single-facility problem is the centroid of the existing facilities. The optimal locations of multiple facilities can be determined by taking partial derivatives of the cost functions and solving the resultant set of linear equations.

The Euclidean distance model, which assumes that the cost is proportional to the Euclidean distance, can be solved by iterative approximation procedures.

An exhaustive treatment of these approaches can be found in Francis and White (1974).

1.3.2 Discrete Space Approach

Initial approaches to the discrete facility location problem were based on heuristic procedures. Kuehn and Hamburger (1963) developed a two-step method for the warehouse location problem. In the first step, warehouse sites are added, one at a time, to the set of existing sites until no further cost savings accrue. At each iteration, a transportation problem is solved to allocate the customers to one of the existing warehouses. In the second step, the warehouses which have become uneconomical during the iterative process, are eliminated and material flows from these warehouses are shifted to other warehouses. Manne (1964) considered the problem in which the warehouses do not have any capacity restrictions. In this case, for a given set of open warehouses, customers can be assigned to the warehouse for which the transportation cost incurred is minimum. However, the large number of possible combinations of open warehouses renders exhaustive enumeration infeasible. Manne's approach is based on starting with an arbitrary feasible set of open warehouses and evaluating all possible one-step movements, i.e., closing one of the existing warehouses or opening a new warehouse. The movement which offers maximum savings is selected. The algorithm terminates when no further cost saving is possible.

Most of the optimum-seeking methods for this problem are based on mixed integer programming formulations. A typical formulation, suggested by Hax and Candea (1984) is given in Appendix 1. In the branch and bound procedure developed by Efroymson and Ray (1966), each node is associated with a potential site for a facility. The solution method is based on Land and Doig's algorithm. Khumawala (1972) modified this approach by proposing efficient branching rules, and simplified the formulation so that the optimal solution at each node could be obtained merely by inspection. Erlenkotter (1978) considered the problem in which the facilities had no capacity restrictions. His procedure is based on

solving the dual problem and obtaining the primal solution by using the complementary slackness theorem. Geoffrion and Graves' (1974) procedure is based on Benders decomposition. For the set covering and the set partitioning formulations of the facility location problem, the reader is referred to Francis and White (1974) and Hax and Candea (1984).

1.4 Aggregate Planning

Facilities location and major capacity questions are usually faced at irregular intervals in an organization's life cycle as it adds or deletes products, as its production volume changes dramatically, as new markets develop or existing ones contract, or as technology alters the economies of its basic production processes. Within shorter time intervals, say six months to two years (but at least a seasonal cycle), the firm must develop ongoing planning procedures to match effectively its existing resources to varying, but perhaps fairly predictable demands being placed on it. The first step in this repetitive planning process is called aggregate planning. Since it is usually difficult to consider all the details of the production process for such a long time-horizon, decisions at this level are based on aggregate information, such as generic units of output, manhours, etc.

Management has two basic options to meet demand fluctuations during a seasonal cycle. It can decide either to follow the demand pattern closely by varying the production rate accordingly, or to maintain a relatively constant production rate and hold inventories and/or permit backorders to absorb the fluctuations. The option of altering the rate of production involves a combination of change in work force size, and introduction of overtime and subcontracting. The relevant costs to be considered in making these decisions are basic production cost, cost of hiring and laying off employees, overtime cost, inventory carrying and backordering costs, and subcontracting costs.

Mathematical modeling approaches to the aggregate planning problem can be classified

into : i) Linear Programming models, ii) Fixed Cost models, and iii) General models, which will be briefly discussed in the next three sections.

1.4.1 Linear Programming Models

Under the assumptions of linear costs, linear constraints, and no setup costs, the aggregate planning problem can be formulated as a linear programming problem. Johnson and Montgomery (1974) presented three such formulations. A goal programming formulation is given in Hax and Candea (1984).

In addition to their computational efficiency, these models provide useful information on the shadow prices of the constrained resources and they are amenable to parametric and sensitivity analysis. Hax and Candea (1984) indicated that the assumptions of linearity may be less restrictive than they appear as production costs may be linear in the range of interest and piecewise linear approximation can be used to deal with separable convex costs.

1.4.2 Fixed Cost Models

Several researchers have looked at the problem in which fixed startup cost and setup time are associated with each run of a production batch. A typical fixed cost model is shown in Appendix 2.

Wagner and Whitin (1958) developed a dynamic programming algorithm for the uncapacitated version of this problem for a single product. They also dealt with the characteristics of the optimal production policy. Zangwill (1966a, 1966b) extended these results for the more general case of concave production costs and piecewise concave inventory carrying and backordering costs. In a later study, Zangwill (1969) developed a procedure which permitted multiple stages of manufacture, each stage being constrained to

have only one preceding and only one succeeding stage.

Manne's (1958) procedure relaxes 0–1 constraints in the original problem to obtain a linear programming problem. If the number of periods in the planning horizon is much larger than the number of products, the optimal solution of the relaxed problem contains very few fractional variables and it can be accepted as a good approximation to the optimal solution. In this approach, however, the number of columns grows exponentially with the number of periods in the planning horizon. Dzielinski and Gomory (1965) circumvented this problem by using a column generation approach. In their method, the original problem is decomposed into subproblems which are solved by the Wagner-Whitin algorithm. The solutions to these subproblems generate the new column to be appended to the existing columns. Lasdon and Terjung (1971) developed a similar column generation approach and used generalized upper bounding techniques to solve the resultant problem at each iteration.

1.4.3 General Models

Holt, Modigliani, Muth, and Simon (1960) decomposed the production costs into four components: i) regular time cost, which is linearly related to the size of the work force, ii) hiring and laying off cost, which is a quadratic function of the change in the work force level, iii) overtime cost, which is zero up to 100% utilization of the work force, and beyond this level, is a quadratic function of the rate of production and the size of the work force, and iv) inventory carrying and backordering cost, which is a quadratic function of the inventory and backordering levels. The formulations proposed by them requires the minimization of the sum of these costs subject to the constraints on material balance from one period to another. By associating Lagrangean multipliers with these constraints and solving the resultant unconstrained problem, optimal solutions for the production rate and work force level for each period can be derived as linear functions of the work force and the

inventory levels during the previous period and the demand forecasts for the future periods.

Jones (1967) proposed a heuristic approach which generates linear decision rules to determine the production and work force levels for any given period. Four parameters are required to define these rules fully. Jones proposed a search method to estimate these parameters. For a given production cost structure, the set of optimal parameters yields the minimum cost for the planning horizon.

The Search Decision Rule proposed by Taubert (1968) treats the aggregate planning problem as a multi-stage decision making process, each stage representing one period in the planning horizon. The inputs to any one stage are i) the initial state of the system, which can be expressed in terms of the inventory level, size of the work force, etc., ii) the production parameters, such as the production cost structure, demand forecast, etc., iii) the set of possible decisions. Each decision is associated with a unique set of outputs in terms of the returns and the state of the system at the end of the stage. The optimum aggregate plan is obtained by searching over all possible decisions at each stage and selecting the set of decisions which maximizes the returns over the planning horizon. For large problems, Taubert suggested that this search routine should be nested within a branch and bound procedure.

1.4.4 Hierarchical Planning Approach

Most of the aggregate planning methods discussed so far have attempted to include all the products over the planning horizon in a single formulation. This approach imposes a heavy computational burden on the model and renders it susceptible to demand uncertainties and forecasting errors. The hierarchical planning approach suggested by Hax and Meal (1975), Bitran and Hax (1977), and Bitran, Haas, and Hax (1981, 1982) circumvents these pitfalls by decomposing the original problem into subproblems at

different levels of aggregation. To carry out this decomposition, they identified three different levels in the product structure: *items* are the end products delivered to the customers, *families* are groups of items which share a common production setup cost, and *types* are groups of families which have similar demand patterns and production costs per unit time.

The first stage of the hierarchical planning process requires aggregate planning of product types. Hax and Meal (1975) proposed a linear programming formulation for this problem which ignores the setup costs. Since this model considers the aggregate demands of product types, the impact of forecasting errors is greatly reduced.

At the second stage, the product types are disaggregated into product families. Bitran and Hax (1977) formulated the disaggregation of each product type as a continuous knapsack problem which minimizes the total setup cost among product families subject to the constraints that the sum of their production quantities should equal the optimal quantity of the product type determined at the first stage. Additionally, these quantities are required to satisfy lower and upper bound constraints determined by the safety stock, the overstock limit and the available inventory.

At the third stage, the product families are disaggregated into items. The objective at this stage is to ensure that the runout times of individual items is the same as the runout time of the product family they belong to, since this would imply that the production requirements of all the items within a family would be generated simultaneously; fewer setups would, therefore, be required in the future. Bitran and Hax (1977) constructed a convex knapsack problem to meet this objective, the constraints being similar to those proposed at the previous stage.

Bitran, Haas, and Hax (1982) extended this approach for a two-stage production system in which the production of parts is decoupled from the assembly of finished goods.

1.5 Scheduling

The lowest level in the prodution planning and control process is scheduling. It is the lowest level in that it is carried out at lower levels (usually shop level) of the organizational hierarchy where current relevant detailed information is available about the status of the machines, workers and production orders. It is also characterized by the fact that planning and execution are carried out repetitively within relatively short time horizons (typically days, hours or minutes) utilizing considerable details about product requirements (tooling, materials, operation sequences, etc.). It is this scheduling problem which is the focus of this research: Conceptualizing and operationalizing a computer-based process which will plan, execute and control the movement and operations of parts, tools, and associated equipment in the flexible manufacturing center at the National Bureau of Standards (AMRF – Automated Manufacturing Research Facility).

To facilitate this effort, and to avoid reinvestigating what has already been accomplished in scheduling research, the bulk of this document reviews relevant scheduling literature results which potentially are applicable to scheduling a flexible manufacturing system.

The scheduling problem can be fully specified by defining it along the dimensions of order characteristics – which has been discussed earlier, job characteristics, processing environment and scheduling objective. In the scheduling literature, the conventional processing environment has been decomposed into four major categories – single stage single machine, single stage parallel machines, flow shops and job shops. This will now be examined in sequence.

The single stage single machine problem is the simplest and the most tractable of the scheduling problems. In this case all jobs require only one operation which has to be performed on a single machine. Though it is rare to see such a system in practice, single machine problems have been studied primarily to provide insight into more complex

production systems. Good heuristics for such systems could quite conceivably be derived from optimum-seeking procedures for the single-machine case. In many instances, sequences based on the analysis of the single machine case have proved to be viable starting points for resolving manufacturing operations involving a bottleneck machine. Further, any group of machines, viewed at a sufficient level of aggregation, could be treated as a single facility.

The single stage parallel machine problem involves scheduling various jobs which require a single operation on a set of machines which are capable individually of performing that operation. This problem could be further subdivided into the classes of identical machines, uniform machines and unrelated machines, based upon the relative efficiencies of the machines. The processing times for any operation are the same for all the identical machines. For uniform machines, these times are proportional, whereas unrelated machines have independent processing times for the same operation. The study of parallel machines is of special significance in an automated manufacturing environment where multiple routing possibilities exist.

In a flow shop, the jobs go through multiple stages of manufacture where each stage is served by a single machine. All the jobs have to go through the same sequence of manufacture though their processing times at individual stages could vary.

Finally, the most general case is that of a job shop where the jobs go through multiple stages of manufacture in random but predetermined sequences. The routes followed by different part types are in general independent of one another. A job shop represents the most flexible use of a manufacturing facility. Unfortunately, the scheduling problems associated with it are extremely hard to solve.

In the context of a manufacturing environment, it is appropriate to introduce the distinction between *deterministic* and *stochastic* scheduling and between *dynamic* and *static* scheduling. In deterministic scheduling, the parameters governing the scheduling problem, e.g., processing times, etc., are known with certainty, whereas for the stochastic case

these parameters are random variables with some known distribution.

In static scheduling, the scheduling problem is fully defined *ab initio*; there is no need for any additional data during the course of manufacture. In contrast, in a dynamic environment, the information required is only partially available in the beginning. Orders arrive at different times in the future which may or may not be known with certainty. Most of the mathematical models presented in the literature are based on the static problem, though some work has been done on the simulation of dynamic job shops.

Two other factors associated with the processing environment involve consideration of job preemption and processing of a job on more than one machine. We need to distinguish between jobs which can be preempted without any additional 'cost' to the schedule and those which cannot be split, i.e., once they are started, they must be completely processed without interruption. Literature on machine shop scheduling has focussed mainly on problems where preemption is not permitted, since this is more reflective of real-world conditions.

Aside from the work done on assembly line balancing, most of the researchers have considered a job to be a single entity, thus prohibiting simultaneous processing at more than one machine. While multi-component jobs can be converted to single component jobs, such a conversion is computationally inefficient.

Scheduling objectives could be related to the facility or to the product. The former category of measures is based on considerations such as machine utilization, idle capacity, set-up time, etc. Scheduling criteria related to the product could be further subdivided into those which are flow time based and those which are due date based. Common flow time based measures include maximum and average flow time, maximum and mean completion time, average work-in-process, maximum production rate, etc. Due-date based measures include maximum and mean lateness and tardiness, number of tardy jobs, etc.

Major survey articles on this subject include the papers by Moore and Wilson (1967), Elmaghraby (1968), Bakshi and Arora (1969), Day and Hottenstein (1970), Panwalker

and Iskander (1977), Salvador (1978), Eilon (1978), Schrage (1979), Graham, Lawler, Lenstra and Rinnooy Kan (1979), Graves (1981), Lawler, Lenstra and Rinnooy Kan (1982) , Lenstra and Rinnooy Kan (1983) and Sen and Gupta (1984). In addition, the books by Muth and Thompson (1963), Conway, Maxwell, and Miller (1967), Eilon and King (1967), Elmaghraby (1973), Baker (1974), Rinnooy Kan (1976), Coffman (1976), Lenstra (1977) and French (1982) provide excellent treatises on this subject.

1.6 Scope of the Review

This paper covers deterministic scheduling for open shops where all production orders are based on specific customer orders. The area of preemptive scheduling has been omitted as an FMS environment would normally preclude job splitting. The paper also reviews the research done in the area of assembly line balancing since it is possible to treat an FMS, or components of it, as a dedicated flow shop for repetitive discrete manufacture. The recent, but limited, literature on the design, planning and scheduling of flexible manufacturing is similarly covered. Though the review does identify the complexity of specific problems, results arising from the theory of NP-completeness have been omitted. Readers interested in these results are referred to Coffman (1976), Rinnooy Kan (1976) and Lenstra (1977).

The paper is organized as follows. The rest of Section 1 covers the basic notations and the problem classification format to be used in the remainder of the paper. Sections 2 through 5 cover the areas of single-machine, parallel machines, flow shop and job shop, respectively. Within each section, the problems are listed according to the scheduling criterion and problem characteristics. Discussion of the assembly line balancing problem is presented in Section 6. Section 7 reviews some of the scheduling research done specifically on flexible manufacturing systems..

1.7 Notations and Problem Classification Format

Notations used in this paper closely follow the normally accepted pattern. The list of these notations is given in Appendix 3.

In the subsequent sections, individual problems have been identified in the n/m/A/B format where,

n : number of jobs

m : number of machines

A : processing environment

B : scheduling criterion

For example,

$1/m//\bar{T}_w$ represents the problem of minimizing average weighted tardiness in a single-machine case,

$n/m/P/C_{max}$ represents the problem of minimizing maximum completion time for a permutation sequence in a general flow shop situation,

$n/m/U/\bar{F}_w$ represents the problem of minimizing average weighted flow time for a set of uniform parallel machines.

2 SINGLE MACHINE

2.1 Minimization of the Average Performance Measure

2.1.1 Mean Flow Time Problems ($n/1//\bar{F}_w$)

The simplest version of this problem for mean unweighted flow time, $n/1//\bar{F}$ is solved by the well known SPT rule which orders the jobs by non- decreasing processing times, i.e., the jobs are ordered as below :

$$p_{(1)} \leq p_{(2)} \leq \ldots \leq p_{(n)},$$

where $p_{(i)}$ is the processing time for the job in the i th position in the schedule. This result can be extended to account for strings, when a set of jobs have to be done together. If $n_{(i)}$ and $p_{(i)}$ denote the number of jobs and the total processing time for all the jobs in the string, the optimal ordering is given by

$$p_{(1)}/n_{(1)} \leq p_{(2)}/n_{(2)} \leq \ldots \leq p_{(k)}/n_{(k)}$$

Mean weighted flow time can be minimized by ordering the jobs according to non-decreasing weighted processing times p_i/w_i. These results are due to Smith (1956).

In the above cases the processing times include the set-up times as well, under the assumption that the latter is independent of the sequence in which the jobs are processed. If this is not true, the problem is NP-complete as it can be shown to be equivalent to the traveling salesman problem. Baker (1974) treats this issue in detail.

The problem assumes dimensions of exponential complexity even in the presence of general precedence constraints as shown by Lenstra and Rinnooy Kan (1978). However, efficient procedures exist for special cases. Horn (1972) and Sidney (1975) proposed O(n logn) algorithms for the case where the precedence constraints form a tree, i.e., each job has at most one predecessor or one successor. Lawler (1978) solved this problem for the series-parallel precedence constraints. Adolphson's (1977) algorithm considers the most general case which covers many special cases of precedence relationships.

The dynamic version of the problem is NP-complete even in the absence of precedence relationships and sequence dependence set-up times. For an exhaustive discussion of these results, the readers are referred to Rinnooy Kan (1976).

For the problem of minimizing weighted completed time, Hariri and Potts (1983) proposed an implicit approach in which they relax the feasibility constraints which require the completion time of any job to be at least as large as the sum of its ready time and its processing time. The bounds are obtained by allowing preemption and applying the dynamic version of Smith's rule. Potts and Van Wassenhove (1983) considered the problem of minimizing weighted completion time for the case in which all jobs have to be

necessarily on time. Their approach is based on Lagrangean relaxation of the deadline constraints. Bianco and Ricciardelli (1982) proposed a similar approach though they use elaborate dominance conditions to prune the size of the enumeration tree.

2.1.2 Mean Tardiness Problems ($n/1//\bar{T}_w$)

As shown in Rinnooy Kan (1976),even the simplest case, where all weights are equal, is NP-complete. However it can be easily proved that if the EDD sequence does not produce more than one late job, it is optimal. If all jobs are late or all jobs have equal due dates, the SPT sequence is optimal. Proofs for these results can be found in Baker (1974). Emmons (1969) developed dominance properties under which one job would precede another in an optimal schedule. Srinivasan (1971) utilized Emmons' dominance properties in his hybrid approach based on dynamic programming to curtail the size of enumeration. Wilkerson and Irwin's (1971) 2-phase heuristic, which is based on schedule construction through pairwise job interchange, does generate good solutions fairly rapidly and assures at least a local optimum. Lawler (1977) proposed an algorithm which solves this problem in $O(n^4 \sum p_j)$ time.

Most of the approaches for the general case have been based on implicit enumeration. Fisher (1976) formulated a Lagrangean problem by associating multipliers with each time period. A lower bound is obtained by solving this modified problem for fixed multipliers. The resultant bound is sharp enough to render even large problems, up to 50 jobs, fairly tractable. Rinnooy Kan, Lageweg and Lenstra (1975) determined acyclic precedence relationships among the jobs and then proceed to establish lower bounds by a linear assignment relaxation of the original problem. Picard and Queyranne (1978) formulated this problem as a time dependent traveling salesman problem – the objective of minimizing mean weighted tardiness is equivalent to finding the shortest path in the network.

Baker and Schrage (1978) proposed an efficient dynamic programming approach to this problem. They defined feasible subsets of jobs, i.e., the set consisting of a job and all its predecessors, and determined the job which when processed last in this subset will produce the minimum tardiness. The precedence relationships are based on the dominance conditions developed by Emmons. They also proposed an efficient labeling method for the feasible subsets to reduce computer storage requirements. Sen and Austin's (1980) algorithm efficiently enumerates all the undominated schedules based on the properties developed by Emmons and by the authors themselves, and has polynomially bounded storage requirements. However the algorithm has variable behavior depending upon the problem, though on the average it is comparable with Baker and Schrage's method.

Comparative studies of various algorithms are available in Baker and Martin (1974), Van Wassenhove and Gelders (1978), Schrage and Baker (1978) and Sen and Austin (1980).

2.1.3 Mean Number of Tardy Jobs Problems ($n/1//\tilde{U}_w$)

Moore (1968) solved the unweighted version of this problem by ordering the jobs in an EDD sequence first and scheduling jobs from the top of this list. The job which has the maximum processing time among those preceding the first late job is removed and placed among the tardy jobs. Sidney (1973) extended this algorithm by considering the case where some jobs have to be on time.

The weighted problem remains NP-complete. Lawler and Moore (1969) proposed a dynamic programming algorithm solvable in $O(n \sum p_j)$ time. Sahni (1976) proposed an approximation algorithm with an error bound of 1-1/k. Kise, Ibaraki and Mine (1978) dealt with a special case for which optimal solutions are available in polynomial time. They looked at the $n/1/r_j/\tilde{U}$ problem where $d_i \leq d_j$ implies $r_i \leq r_j$. The fully polynomial approximation scheme of Gens and Levner (1981) is based on deriving the lower bounds by

solving the associated knapsack problem.

Precedence constraints produce NP-completeness even for the unweighted version of the problem. Ibarra and Kim (1978) suggested an approximation algorithm for the weighted problem with tree precedence relationships with an error bound of 1–1/k.

2.2 Minimization of the Maximum of the Performance Measures

2.2.1 Maximum Completion Time Problems ($n/1//C_{max}$)

For the simplest version of this problem, $n/1//C_{max}$, any feasible schedule is optimal. For the case of non-zero ready times, the optimal schedule requires ordering the jobs according to non-decreasing ready times. Introduction of sequence dependence, however, introduces exponential complexity. The resultant problem is similar to the traveling salesman problem and needs to be solved in a similar manner. White and Wilson (1977) proposed a heuristic which exploits the fact that the set-up time can be broken down into its constituent elements. The next job to be processed is then the one which utilizes the element which is the most time consuming one.

Lawler (1973) suggested a polynomially bounded algorithm which minimizes the maximum of arbitrary non-decreasing cost functions in the presence of general precedence constraints. The dynamic version of this problem however remains NP-complete.

2.2.2 Maximum Lateness Problems ($n/1//L_{max}$)

Since tardiness is always less than or equal to lateness, any algorithm which minimizes maximum lateness will necessarily be optimal for the maximum tardiness problem as well, though the reverse is not true. The simplest version of this problem is optimally solved by the EDD sequence, which orders the jobs by non-decreasing due

dates. This rule can be readily extended to account for precedence by modifying the due date to be the minimum of the job's own due date and those of its predecessors.

The dynamic case of this problem however is NP-complete, though efficient algorithms exist for special cases. If all r_j are equal, the EDD sequence is optimal. If all d_j are equal, the jobs need to be ordered according to non-decreasing ready times. If all p_j are equal, Simons (1978) proposed a $O(n^3 \log n)$ algorithm which effectively utilizes the modified Jackson's rule. These special cases are tractable even in the presence of precedence relationships. Several enumeration approaches have been proposed for the general problem. Baker and Su (1974) generated all possible active schedules and provide a lower bound based on the application of the extended Jackson's rule to the problem which allows preemption. McMahon and Florian (1975) started with an initial solution based on Schrage's heuristic and proceeded to develop a tree which has a complete solution associated with each node. Lageweg, Lenstra and Rinnooy Kan (1978) subsequently proposed modifications to this approach which involve inversion of the problem to increase its efficiency. Carlier (1982) formed two branches at each node corresponding to the critical job, one in which this critical job precedes all the rest in the critical set and the other in which it succeeds them. Like Baker and Su, the bounding scheme is based on allowing preemption. Erschler, Fontan, Merce, and Roubillat (1982,1983) developed dominance conditions based on the value of $d_j - r_j$. The jobs with low values turn out to be the critical ones and they are sequenced in the order of non-decreasing r_j. The other jobs are scheduled subsequently. Larson and Dessouky (1978) and Kise, Ibaraki, and Mine (1979) proposed heuristics for the same problem.

2.3 Bicriterion Objectives

Several authors have considered the problem of minimizing the sum of weighted tardiness and weighted flow times. Gelders and Kleindorfer (1974, 1975) used a branch

and bound approach. They showed the lower bounding function to be convex. The bounds were derived from the optimal dual variables of the transportation problem solved at each node. Van Wassenhove and Gelders (1980) determined the points on the efficient frontier through a pseudopolynomial algorithm which is based on the concepts developed by Smith (1956). Van Wassenhove and Baker (1980) considered the objective of minimizing the sum of maximum completion cost and a linear crashing cost. Other works in this area include those of Bansal (1980) and Burns (1976).

3 PARALLEL MACHINES

As mentioned earlier, parallel machines can be categorized as identical, uniform and unrelated. The problem complexity increases as the processing times become gradually more unrelated. Hence any algorithm which is applicable for the more complex problem will solve the simpler problem as well. Hence in this section we look first at the unrelated processor problem and subsequently at the uniform and identical machines in that order.

3.1 Unrelated Machines

Very little work as been done in the area of unrelated processors mainly because the problems assume dimensions of exponential complexity even for relatively simple processing environments. Surprisingly enough, there are very few heuristics to report either.

Rothkopf (1966) formulated an m-dimensional dynamic programming model which is applicable to mean weighted flow time, weighted number of tardy jobs, maximum lateness, maximum completion time and a special case of weighted tardiness where all jobs have the same due date. The computational requirements are however severe.

The problem of minimizing mean flow time, n/R//$\bar{F}$, can be treated as a transportation

problem as shown by Bruno, Coffman and Sethi (1974) and Horn (1973). Introduction of any complexity in the form of weights, sequence dependence or ready times makes this problem NP-complete.

For the $n/R//C_{max}$ problem, Ibarra and Kim (1977) proposed a set of five approximation algorithms which in general tend to schedule the jobs on the machines which process them most efficiently. However the error is bounded only by m. Davis and Jaffe (1981) suggested an adaptation of the list scheduling algorithm with a worst case performance of $O(\sqrt{m})$.

3.2 Uniform Machines

Conway, Maxwell and Miller (1967) have shown that the unweighted version of the problem, $n/U//\bar{F}_w$, can be optimally solved in O(nlogn) steps by generalizing the case for identical machines, which is shown in the next section. Introduction of weights and other complexities makes the problem NP-complete. Horowitz and Sahni (1976) suggested an approximation algorithm for the weighted mean flow time problem.

For the $n/U//C_{max}$ problem Gonzalez, Ibarra and Sahni (1977) proposed an adaptation of the longest processing time algorithm in which the jobs are scheduled in an LPT sequence and they are assigned to the machine on which they would be completed the soonest. The error is bounded by $2-1/(m+1)$. Cho and Sahni (1980) showed that the arbitrary list scheduling algorithms have a worst case performance bounded by O(m). Freisen and Langston (1983) showed that the value of C_{max} generated by the multifit heuristic,which is an adaptation of the First Fit Decreasing approach to the bin packing problem, can be at the most 1.341–1.4 times the optimal value.

As shown in Rinnooy Kan (1976), if all jobs have unit processing times, both $n/U//\bar{F}$ and $n/U//C_{max}$ problems can be formulated and solved as a Transportation Problem.

3.3 Identical Machines

3.3.1 Flow Based Criteria

For the problem $n/P//\bar{F}$, the optimal solution can be generated by first ordering the jobs in an SPT sequence and then assigning them successively to the machine with the minimum load assigned. In a recent work, Meilijson and Tamir (1984) showed that the SPT rule remains optimal if the machine speeds increase over time. If they decrease over time the problem becomes NP-complete. The weighted version of this problem, $n/P//\bar{F}_w$ is NP-complete though Baker and Merton (1973) reported good results with the heuristic which orders the jobs in WSPT sequence and assigns them to the machines in a manner similar to the unweighted version. Eastman, Even and Isaacs (1964) developed the lower bound for the mean flow time based on the minimal values of F_w for a single machine and F_w for n machines. Sahni (1976) proposed a fully polynomial approximation scheme with an error bound of $1+1/k$.

The special case of this problem where all processing times equal 1 can be formulated as a Transportation Problem. However, Rinnooy Kan (1976) has shown that in the presence of precedence constraints, even this problem is NP-complete.

The problem, $n/P//C_{max}$, is NP-complete, though several approximation algorithms have been proposed for it. Graham (1969) showed that a list scheduling algorithm based on LPT ordering produces schedules for which the maximum completion time is at the most 4/3 times the optimal value. This was subsequently improved upon by Coffman, Garey and Johnson (1978) whose Multifit algorithm is guaranteed to generate a schedule with the maximum completion time within 1.22 times the optimal value. Another approximation algorithm has been proposed by Sahni (1976). Leung (1982) presented a dynamic programming formulation of the problem where the processing times can take at most k different values.

Hu (1961) presented an optimal solution for the $n/1/prec,p_j=1/C_{max}$ problem where the precedence relationships form an intree, i.e., each nonterminal node has has only one successor node. For general precedence relationships, Lam and Sethi (1977) and Ibarra and Kim (1976) proposed approximation algorithms.

3.3.2 Due Date Based Criteria

For the weighted tardiness problem, $n/1//T_w$, Elmaghraby and Park (1974) developed a branch and bound approach which uses the lower bound developed by Eastman, Even and Isaacs (1964). Their algorithm has been further refined by Barnes and Brennan (1977) by providing infeasibility tests for fathoming nodes. They report, however, that this approach can be used only for very small problems. Pritsker, Watters and Wolfe (1969) have proposed a 0–1 LP formulation which minimizes makespan as well as total tardiness penalties. Nunnikhoven and Emmons (1977) proposed some approximation algorithms as well as two optimum seeking methods. Root (1965) presented the optimal schedule for the unweighted problem when all the due dates are equal.

The problem of minimizing maximum lateness, $n/1//L_{max}$, has been studied by Horn (1974) and Tilquin (1975). The latter work presented a heuristic solution for the problem with non-zero ready times.

In the presence of precedence constraints this problem is NP-complete even for unit processing times. However, Hu's algorithm can be extended to the case of intrees.

4 FLOW SHOP

Flow Shop problems are characterized by the fact that all the jobs have to go necessarily through all the machines in the shop in the same sequence. No by-passing is permitted though the processing times of the jobs at different machines need not be the

same.

A permutation schedule is one which specifies the same sequence of jobs at all the machines. It can be shown, refer Baker (1974) for a detailed proof, that the set of permutation schedules is dominant for the case of n/2/F/Regular measures and also for n/3/F/C_{max}. However in general for m > 3, it need not contain an optimal schedule. In spite of this, research in the area of Flow Shops has concentrated on the permutation schedules only, mainly because the more general case is computationally intractable. As we shall see even for permutation schedules, only the simplest of problems can be solved efficiently.

Problem complexity has also precluded consideration of performance measures other than maximum completion time and for some special cases, maximum lateness. Processing times have been assumed to be non-sequence dependent and all jobs have been assumed to be available at time zero.

NP-completeness results have forced researchers to look for exploitable problem characteristics in order to be able to generate efficient solutions. Research can therefore be broadly classified into the areas of looking into problems with special characteristics, generation of dominance conditions to prune the size of enumeration, development of strong bounds and finally development of good heuristics. A special case of the flow shop problem has also been considered by researchers where no intermediate storage space is provided.

The most fundamental result for flow shop problems has been provided by Johnson (1954) for the n/2/F/C_{max} problem. If p_{i1} and p_{i2} are the processing times for job i on the two machines, then in an optimal schedule i precedes j if,

$$\min (p_{i1}, p_{j2}) \leq \min (p_{i2}, p_{j1})$$

Several variants of this result have been used for generating solutions for flow shops with special structure. Johnson (1954) considered the case for n/3/F/C_{max} where the second machine is a non-bottleneck one, i.e.,

$\min (p_{i1}) \leq \max (p_{i2})$, or

$\min (p_{i3}) \leq \max (p_{i2})$

Burns and Rooker (1978) have shown that this requirement can be simplified to

$\max (p_{i2}) \leq \min (\min (p_{i1}), \min (p_{i3}))$

Arthanari and Mukhopahyay (1971) proposed an optimization algorithm based on solving n n/2/F/C_{max} problems for the case where

$\max (p_{i1}) \leq \min (p_{i2})$, or

$\max (p_{i3}) \leq \min (p_{i2})$.

Szwarc (1978) considered the case where i) $p_{i2} = p_{j2}$ and, ii) if $p_{i1} \leq p_{j1}$ then $p_{i3} \geq p_{j3}$, for all i and j. Achugbue and Chin (1982) looked at the problem where either $p_{i1} \geq p_{i2} \geq p_{i3}$ or $p_{i1} \leq p_{i2} \leq p_{i3}$, for all i.

Smith et al (1975) defined an ordered flow shop to be one where the following conditions hold

1) If $p_{it} \geq p_{kt}$ for i,k ϵ N, then

$p_{ij} \geq p_{kj}$ for all j ϵ M.

2) If $p_{ij} \geq p_{is}$ for j,s ϵ M, then

$p_{kj} \geq p_{ks}$ for all k ϵ N.

For this problem they proposed a branch and bound algorithm. Panwalker and Woollam (1979) showed that the n/m/P/$\bar{C}$ problem is optimally solved by the SPT sequence for the ordered flow shop case. This result was extended by Panwalkar and Khan (1976) for the case where no intermediate storage space is provided. For the C_{max} problem, Panwalker and Woollam (1980) showed that if the maximum processing time is at machine 1, then LPT schedule is optimal; if the maximum processing time is on machine m then SPT schedule is optimal. For the general case a sequence (σ, σ') would be optimal where σ is ordered in an SPT sequence and σ' in an LPT sequence. Chin and Tsai (1981) proposed polynomial algorithms for some special cases of ordered flow shops and have also

given bounds for the length of arbitrary permutation schedules.

Yoshida and Hitomi (1979) showed that for the $n/2/r_i/C_{max}$ problem Johnson's rule can be modified to state that job i would precede job j in an optimal schedule if,

$$\min (r_i - r_j + p_{i1}, p_{j2}) \leq \min (r_i - r_j + p_{j1}, p_{i2}).$$

Much of the work in developing dominance conditions has been done by Szwarc (1971, 1973), Smith and Dudek (1967, 1969) and McMahon (1969). If σ refers to a partial schedule wherein the first s positions are occupied by $\sigma(1),\sigma(2),...\sigma(s)$ and $C(\sigma, k)$ is the completion time of σ on machine k, then σ_1 is said to dominate σ_2 if $C(\sigma_1, k) \leq C(\sigma_2, k)$. In particular, attention has been paid to the different conditions under which all completions of (σ, j) are dominated by completions of (σ, i, j), in which case j can be eliminated from the s+1st place in the schedule. If we define $\Delta_k = C(\sigma, i, j, k) - C(\sigma, j, k)$, then the results obtained by Szwarc (1971, 1973), McMahon (1969) and Smith and Dudek (1967, 1969) can be used to show that the most stringent condition under which j can be eliminated from the s+1st position is,

$$\text{Max } (\Delta_{i-1}, \Delta_i) \leq p_{jk} \text{ for } k = 1,2,...,m$$

Dominance conditions have however proved to be too restrictive to be of much use in reducing the size of actual problems. Consequently, not much work has been done on them in the recent past.

Rinnooy Kan (1976) has given an excellent summary of the bounds used in different enumeration approaches to this problem. Essentially, the m-machine case can be looked upon as consisting of 2 bottleneck and other non-bottleneck machines. The capacity constraints of all but two of the machines, M_u and M_v, can be relaxed to reduce the problem to a 5 machine case comprising of M_{*u}, M_u, M_{uv}, M_v and M_{v*}, where $1<u<v<m$. M_{*u} can then be seen as setting release dates r_{i*u} on machine M_u and M_{v*} as setting due dates $-q_{iv*}$ on machine M_v, where,

$$r_{i*u} = \max_{1<k<u-1} (C(\sigma,k) + \sum_{s=k}^{(u-1)} p_{is})$$

$$q_{iuv} = \sum_{s=(u+1)}^{(v-1)} p_{is}$$

$$q_{iv*} = \sum_{s=(u+1)}^{(v-1)} p_{is}$$

The problem can then be stated as that of minimizing L_{max} on M_v subject to the release dates r_{i*u} on M_u. The problem can be further simplified by eliminating any of the non-bottleneck machines M_{*u}, M_{uv}, M_{v*} by suitably underestimating its contribution to the total completion time as the minimum processing time on that machine. These contributions can then be added to the optimal solution of the remaining problem to obtain lower bounds.

Ignall and Schrage (1965) and McMahon (1969) considered the case where $u=v$ and M_{*u} and M_{v*} have been removed to yield the machine based bound. A somewhat stronger bound can be obtained by removing only M_{*u} and applying Jackson's rule to the resulting $n/1//L_{max}$ problem.

The best bound currently available has been developed by Rinnooy Kan (1976) who has applied Johnson's rule to the $n/2/P/C_{max}$ problem obtained by removing M_{*u},M_{uv} and M_{v*}. As pointed out by Rinnooy Kan, stronger bounds can be obtained by developing fast algorithms or stronger bounds for the $n/1/r_i//L_{max}$ and/or $n/2/P,r_i/C_{max}$ problems.

Heuristic approaches have been largely motivated by Johnson's algorithm. An efficient procedure has been developed by Campbell, Dudek and Smith (1970) who use Johnson's rule to solve a series of two-machine relaxations to the actual m − machine problem. Palmer (1965) proposed a method by which each job is assigned an index which is determined by the relative processing times at various machines. Gupta (1971) proposed a similar heuristic which assigns priorities to individual jobs based on their relative processing times at successive machines. Dannenbring's (1977) heuristic also uses

Johnson's rule to solve a series of 2 machine relaxations to the original problem. Additionally, he considered pairwise interchange of adjacent jobs to generate as much improvement as possible. King and Spachis (1980) proposed and tested 5 heuristics: the basic thrust of their approach is on reducing the machine idle time while waiting for the next job to be processed. They also contend that idle times on machines farther downstream are more critical than those on the earlier machine. Nawaz, Enscore and Ham (1983) argued that jobs with greater total processing times are more critical and need to be scheduled first. Their procedure lists the jobs in the order of decreasing processing times; partial schedules of increasing cardinality are generated by successively scheduling the job from the top of the list through an exhaustive search procedure.

As mentioned earlier, the no-wait case where the jobs do not have any intermediate storage space has attracted a lot of attention in research. Wismer (1972) and Reddi and Ramamoorthy(1972) showed this problem to be equivalent to the traveling salesman problem. Rinnooy Kan (1976) and Lenstra (1977) gave elaborate NP-completeness proofs for this problem.

Gelders and Sambandam (1978) presented four heuristics for the problem where the sum of weighted tardiness and weighted flow time is to be minimized.

Graves, Meal, Stefak and Zeghmi (1983) studied the case of a re-entrant flow shop and have proposed a heuristic algorithm for it. In such a flow shop a job can come back to a machine it has previously visited.

5 JOB SHOP

Job shops represent the most versatile and the most flexible of all operation systems. These systems typically accomodate several part types – each with arbitrary but predetermined routing of operations to be done at several machines. There are no restrictions on the routes followed by different jobs.

Since the job shop scheduling problem is the most general one, it is also the most intractable of all scheduling problems. Most of the research done in this area has focused on solving the makespan problem. However, as Rinnooy Kan and others showed, even the simplest of problems remains NP-complete. The classic 10-job 10-machine problem posed in Muth and Thompson (1963) remains unsolved.

5.1 Static Job Shop

The general static job shop problem can be represented as a disjunctive graph where the nodes denote the operations to be done with weights equal to their parameters. There are two dummy nodes with zero weights corresponding to the start and the finish of all the jobs. The conjunctive arcs link successive operations to be done on the same job, the start node to the first operation of each job and the end node to the last operation of each job. A pair of disjunctive arcs link two operations which can be performed simultaneously on the same machine. The solution to this problem requires selection of exactly one arc from the pairs of disjunctive arcs such that an acyclic directed graph is obtained which optimizes the scheduling objective.

Enumerative approaches to this problem have varied in the selection of branching rule and determination of the lower bound at each node of the tree corresponding to the selection of a particular disjunctive arc. In most of these enumeration approaches, descendents have been created at each node by generating all *active* schedules for the unscheduled operations. However, Charlton and Death (1970) created two sub-problems at each node corresponding to each of the two disjunctive arcs. The bounding mechanism used by researchers is based on relaxing the capacity constraints on all machines except one, say M_j. The relaxed problem then requires determining, on M_j, the sequence of schedulable operations O_i, with ready times r_{ij}, processing times p_{ij} and due dates $-q_{ij}$ such that maximum lateness is minimized. This relaxed $n/1/prec,r_i/L_{max}$ problem is itself NP-

complete. Consequently, researchers derived lower bounds by ignoring some of the parameters or the precedence relationship itself. Ashour, Chin and Moore (1973), and Ashour and Hiremath (1973) ignored r_{ij} and $-q_{ij}$. The lower bound proposed by them is the sum of the earliest possible starting time of an operation on M_j and the total processing time for all operations on M_j. Ashour and Parker (1971), Charlton and Death (1970), Nabeshima (1971) and Ashour, Moore and Chin (1974) proposed a lower bound equal to the maximum, among all operations scheduled on M_j, of the sum of the starting time, the processing time on M_j and the remaining processing times on all the other machines. The lower bound suggested by Brooks and White (1965) and Florian, Trepant, and McMahon (1971) is the sum of the minimum possible starting time for a schedulable operation on M_j and the optimal solution to the $n/1/r_i/C_{max}$ problem. In the schemes proposed by Bratley, Florian and Robillard (1973) and McMahon and Florian (1975), the lower bound is the optimal solution to the associated $n/1/r_i/L_{max}$ problem. Rinnooy Kan (1976) covered various enumeration approaches in a fair amount of detail.

Fisher, Lageweg, Lenstra and Rinnooy Kan (1983) studied surrogate duality relaxations of capacity and precedence constraints. These constraints have been weighted and aggregated to a single constraint which is strengthened by iterating over all possible weights. Though this procedure involves considerably more computations, the improvement achieved over McMahon and Florian's approach appears to be marginal.

Two other, essentially heuristic, approaches have been proposed for the job shop scheduling problem. The first is based on generating *non-delay* schedules; conflicts arising between jobs being resolved by a specified priority rule. While it can be easily proved that non-delay schedules do not constitute a dominant set, empirical investigations by Jeremiah, Lalchandani and Schrage (1964) showed that they outperformed active schedules for both mean flow time and maximum flow time criteria.

Many priority rules have been discussed in the literature. Panwalker and Iskander (1977) listed over a hundred of them. Of these, six of the most widely studied rules are

shortest processing time, maximum work remaining, least work remaining, maximum number of operations remaining, first come first served and random selection. Jeremiah, Lalchandani and Schrage (1964) found that for makespan problems the maximum work remaining rule yielded the best results whereas for mean flow time problem rules giving priority to jobs with low processing time requirements, like shortest processing time and least work remaining, performed the best. Gere (1966) studied the interaction of different priority rules and heuristics. He concluded that heuristics which anticipate future progress of the schedule performed better; he also found that selection of the right heuristic was more important than choosing the right priority rule for determining the quality of the solution.

The second approach is based on generating a series of schedules, through Monte Carlo simulation, feasible to the problem and selecting the best among them. Though the number of feasible schedules is very large, the number of different makespans is much less. Randolph, Swinson and Ellingsen (1973) presented Bayesian Analysis methods for generating schedules and determining the stopping rules. Elmaghraby (1968) and Rinnooy Kan (1976) also discussed these issues.

Probabilistic dispatching is the technique of generating random schedules based on a hierarchy of priority rules. In their study, Jeremiah, Lalchandani and Schrage (1964) found that, for makespan problems, probabilistic dispatching generated schedules superior to those determined by simple priority rules or by random sampling. Conway, Maxwell and Miller (1967) and Baker (1974) provided detailed discussion on this subject.

5.2 Dynamic Job Shop

In a dynamic job shop, jobs arrive randomly over time and queue up for their first machine(s). Upon machine availability, a job is selected from this queue by using a pre-specified priority rule or a priority index. After the processing is complete, the job is routed

to the next machine in its sequence where the process of queueing and selection is repeated.

In their survey, Day and Hottenstein (1970) identified six parameters for the dynamic job shop scheduling problem: i) job arrival pattern, ii) processing time distribution, iii) shop size, iv) job routing specification, v) priority dispatching rule, and vi) assignment of due dates. Of these, the first four refer to the dimensions of the problem while the last two are the decision variables whose impact on shop performance has been the focus of various simulation studies.

Most of the researchers have assumed job interarrival and processing times to be exponentially distributed. As Baker (1974) noted, there is little evidence to believe that the performance of different scheduling rules is strongly dependent upon the exact distribution of these random variables. Studies conducted by Baker and Dzielinski (1960) and Conway, Johnson, and Maxwell (1960) found the impact of shop size on the relative merits of the priority rules is insignificant. Job routings can be specified as a square transition matrix whose elements a_{ij} represent the probability of a job being routed to machine j after completing its operation on machine i. Different job routings, ranging from the pure job shop where a job is equally likely to visit any of the remaining machines to pure flow shop where there is a fixed sequence to be followed by all the jobs, have been examined.

Baker (1974) classified priority rules along two dimensions. A rule could be local, when it utilizes information about the jobs only on the machine to which the job has to be assigned, or global, when it examines queues at other machines as well. In addition, a rule could be static – if the priority assignment to a job does not change over time or dynamic, if it does.

When a job enters the shop, it may either have a due date set exogenously, or it may be assigned a due date. Various due date assignment rules have been investigated. Baker (1984) classified these rules into the categories of allowance based rules, slack based rules and ratio based rules.

Bertrand (1983) has shown that due date assignment rules based on time phased workload and capacity information can significantly reduce lateness variance. The parameters he varied were the minimum allowance for waiting and the maximum fraction of the remaining shop capacity available for loading. He concluded that if these parameters were set 'properly', mean lateness and lateness variance were independent of the number of operations in a job. Also, within a range of these parameter values, the shop performance did not change very much. Weeks' (1979) study also revealed that due date assignment rules based on individual job flow time conditional upon shop congestion, performed better than rules based only on total work content. Miyazaki (1981) proposed a somewhat complicated due date assignment rule which incorporates mean and standard deviation of waiting time per operation. In his investigation, priorities based on S/OPN performed better than FT/OPN for the measures of N_t,$\bar{T}$, and T_{max} especially when the due dates were very tight. Given the job due date, individual operations can be assigned due dates by dividing the total flow time into segments, one for each operation. These segments can be formed in a number of ways. Kanet and Hayya (1982) found the TWK rule to be superior to the CON rule for determining operation due dates and the performance of operation based due date rules was superior to that of job based due date rules.

Simulation studies have compared various priority dispatching rules on mean flow time as well as due date based measures. In his comprehensive study, Conway (1965a) found SPT to be superior to all the other rules for minimizing mean flow time. Nanot (1963) reached a similar conclusion and also found that the result held for different routing matrices and different shop loads. Conway reported further improvement in the performance of the SPT rule when it was used in combination with other rules such as WINQ. In another study, Conway (1965b) compared these priority dispatching rules for the measures of proportion of jobs tardy, mean lateness and lateness variance. For the case of pre-assigned due dates, he found that the S/OPN rule had the lowest proportion of

jobs tardy and minimum lateness variance. The SPT rule provided a low proportion of jobs tardy, though it did not perform very well on the other two measures. Conway also found that though in general, the performance of all the rules deteriorated under increased workloads, the SPT rule was the least sensitive to it. The study did reveal a possible crossover point; at relatively low shop utilization levels, the S/OPN was superior while at higher levels, the SPT rule was preferable. Carroll (1965), in his investigation for the mean tardiness measure, found that the COVERT rule consistently outperformed the SPT and the S/OPN rules at varying shop load levels and under different flow allowances. Baker and Bertrand's (1981,1982) study revealed that when due dates were lax, EDD dispatching rule was the best for mean tardiness while the SPT rule was superior when due dates were very tight. In the intermediate range, the MDD rule produced the best results and it was fairly robust with respect to due date tightness. Kanet and Hayya (1982) indicated the superiority of operation based priority rules as compared with job based rules. In their investigation for the measures of mean lateness, lateness variance and proportion of jobs tardy, the OCR rule yielded the best results. Elvers and Taube's (1983) study also indicated the superiority of the SPT rule at high levels of shop utilizations for the objective of minimizing proportion of jobs tardy. They also found that in most of the cases, assuming stochastic processing times does not lead to stronger results.

Baker (1984) conducted a detailed investigation of the interactions between due date assignment rules, priority dispatching rules, shop utilization levels and due date tightness. He first compared different operation due date assignment rules under varying job due date assignment conditions and found the TWK rule to be superior as well as robust. This rule was used subsequently to set operation due dates. In a three-way study of job due date assignment rules, priority dispatching rules and due date tightness, he found the TWK rule to be the best for job due date assignment as well, except when the tardiness levels were already very low. His experiments also demonstrated that the performance of dispatching rules was strongly dependent upon due date tightness. For

relatively loose due dates, nearly all the rules, with the exception of the SPT rule, performed equally well for the measures of mean tardiness and proportion of jobs tardy. However, as due dates became tighter, there was a rapid divergence in the performance of these priority rules. Overall performance of operation based due date rules was better with the MOD rule yielding the lowest values of mean tardiness at moderate to very high values of due date tightness. The SPT rule however, resulted in the lowest proportion of jobs tardy and was quite robust for the mean tardiness criterion as well.

Hottenstein (1970) studied the impact of job expediting. He allowed jobs to be expedited when their slack became negative or when their due dates were revised. He found that expediting adversely affected overall mean flow time and proportion of jobs tardy, especially at high shop utilization levels.

6 ASSEMBLY LINE BALANCING

Closely associated with the theory of flow shop scheduling is the issue of assembly line balancing. In an assembly line, a set of ordered repetitive tasks need to be assigned to a set of workstations such that the total operating time at any station is less than the cycle time. The obvious practical usefulness of this topic has generated a fairly rich literature. It is possible to classify research done in this area along the dimensions of deterministic versus stochastic problems – depending upon the variability of processing times, and single model versus mixed model problems – contingent upon whether the tasks relate to a single product type or otherwise.

In accordance with the general approach adopted for scheduling problems, this paper covers only deterministic ALB problems. For this category of problems however, all other dimensions are covered. Both optimum-seeking and heuristic approaches are discussed.

6.1 Single Model Problems

6.1.1 Optimization Approaches

Most of the optimum-seeking approaches to the ALB problems have been based on integer programming formulations. In Bowman's (1960) method, as modified by White (1961), the assignment of tasks to workstations, has been expressed as 0-1 variables. There is a progressive increase increase in the cost of each additional workstation created beyond the theoretical minimum number. The constraints require the total processing time at each workstation to be less than or equal to the cycle time, the ordering of tasks according to the precedence relationships, and assignment of every task to exactly one workstation. Thangavelu and Shetty (1971) proposed an alternative formulation which reduces the number of constraints substantially. They applied Balas' enumerative approach, strengthened by conditional feasibility tests to reduce computational requirements. The initial upper bound is based on the solution generated by Helgeson and Birnie's (1961) heuristic method. Charlton and Death (1970) used the Bowman-White formulation for their branch and bound approach. Each node in their tree represents a partial assignment of tasks, and lower bounds are derived by adding the total processing time to the total idle time incurred so far. Patterson and Albracht (1975) incorporated the concept of the earliest and the latest feasible stations for each task. They eliminated the assignment constraints by recognizing them to be multiple choice constraints. In their approach, conditional feasibility tests are used for precedence constraints and binary infeasibility tests are used for cycle time constraints. They also proposed a Fibonacci search procedure for determining the number of the workstation to which the final tasks need to be assigned for the purpose of deriving the upper bound.

Van Assche and Herroelen (1979) proposed a frontier search approach where each node in the tree represents the task assignment to a particular workstation. The lower

bound associated with each node is the sum of the number of workstations already generated and the theoretical minimum number of workstations still to be created. The average processing required at each of these future workstations is the penalty associated with the node which governs the branching process. Van Assche and Herroelen also proposed heuristic tie-breaking rules to augment the branching mechanism, and dominance tests to expedite fathoming.

Wee and Magazine (1981) developed a branch and bound algorithm which relies on heuristically determined upper bounds. The heuristics used by them are adaptations of the first fit decreasing rule for bin packing problem and Helgeson and Birnie's Ranked Positional Weight method. Each node in the tree represents a feasible task assignment to a workstation. At node i, descendents are created corresponding to all possible maximal assignments to workstation (i+1). Two heuristics, one based on the first fit decreasing rule for bin packing problem and the other based on Helgeson and Birnie's Ranked Positional Weight method, have been proposed for determining the upper bound. The lower bound is the sum of the number of workstations already generated and the theoretical minimum number required for the remaining processing time. Branching favors the node with the minimum cumulative idle time.

Talbot and Patterson (1984) have proposed an enumeration approach which is based on an adaptation of the Balas additive algorithm and which exploits the problem structure to expedite backtracking and fathoming. Integer variables, equal to the number of tasks, have been used to reduce computer storage requirements significantly. The order in which the variables are considered for augmenting is pre-specified by a node numbering scheme. They also use the concept of earliest and latest feasible station to limit the range of variable values. 'Chains' and 'network cuts' have been used for rapid fathoming and backtracking. A chain is a set of consecutively numbered tasks such that for each task the immediate successor is the task which follows it in the chain. A workstation s is called a network cut if, i) there is at least one task t for which the earliest feasible workstation is

(s+1), and ii) the earliest feasible workstations for all tasks t", t">t, is greater than s. These cuts essentially decompose the original problem into relatively independent sub-problems; improvement in one sub-problem does not improve solution efficiency with respect to other sub-problems. With the use of chains and network cuts, backtracking can proceed directly to the highest numbered task not in the current chain and the highest numbered task which has been currently assigned a workstation numbered higher than its cut, thereby eliminating a considerable amount of search. Good partial schedules are saved at each cut to expedite fathoming. Talbot and Patterson also use idle time tests, based on solving associated knapsack problems, at each cut for rapid fathoming.

The 'best bud search' procedure constructed by Nevins (1972), is essentially a branch and bound method which starts with an upper bound N_{ub} on the number of stations to be generated. Each node represents a subset of tasks which have been assigned to the first i stations. The sub-problem associated with the node is to assign the remaining tasks to $N_{ub}-i$ stations. A descendent of any node adds new tasks to the ones assigned by the parent node. These new tasks constitute a workstation. The branching rule selects the descendent with the minimum average processing time to be assigned to the remaining workstations. If a solution is found for N_{ub} stations, the upper bound is reduced to $N_{ub}-1$ and the process is repeated.

Held, Karp and Sharesian (1963) proposed a dynamic programming approach which is based on determining the optimal cost for each feasible subset – a subset of tasks which have no predecessors. The cost of any feasible subset is the sum of the number of stations required for that subset and the total time required at the last workstation. The costs of these feasible subsets with increasing cardinality is then computed recursively. More recently, Schrage and Baker (1978) proposed an efficient dynamic programming approach in which they generate feasible subsets based on a lexicographic ordering scheme and associate unique labels to these feasible subsets.

Klein (1963) developed a method which requires solving a series of assignment

problems for minimizing idle time associated with a feasible sequence of tasks. Gutjahr and Nemhauser's (1964) procedure involves solving the shortest path problem in a network where the nodes correspond to the feasible subsets and the directed arcs (i,j) link feasible subsets S_i and S_j such that S_i is a subset of S_j. These arcs then represent feasible assignments to a workstation.

6.1.2 Heuristic Approaches

Baybars (1984) categorized heuristic methods into four classes: i) rank and assign methods, ii) enumerative methods, iii) sampling methods, and iv) other approximation methods.

Rank and assign methods implement a list processing algorithm in which the priority rankings of tasks are generated by the attributes of these tasks. In Moodie and Young's (1965) Largest Candidate Method, the tasks are processed in the descending order of their processing times subject to the precedence relationships and the cycle time constraints. New workstations are generated when the processing time for the next available task exceeds the idle time remaining at each workstation. Helgeson and Birnie (1961) determine the positional weight for each task, defined as the sum of the processing times of that task and its successors. Assignment of tasks is done in the descending order of the positional weights without violating feasibility. Kilbridge and Wester (1961) construct a precedence diagram in which the tasks with identical precedence are positioned vertically in columns. If a task can be positioned in more than one column, its transferability is recorded. The assignment of tasks to workstations starts from column 1 and progresses successively to other columns. Talbot, Gehrlein and Patterson (1985) listed thirteen different ways of generating priority rankings.

Hoffman's (1963) enumeration method considers workstations sequentially, and assigns to each a combination of tasks which minimize the total idle time. The optimum set

of tasks to be assigned is determined by complete enumeration. The precedent feasible set of remaining tasks is updated at each workstation. To implement the enumeration process, Hoffman uses a precedence matrix and an index vector. The algorithm provides for balancing in the backward direction as well, if necessary. Gehrlein and Patterson (1975) proposed modifications to the original procedure whereby the search for optimal assignment of tasks is curtailed if the idle time realized is 'acceptably' close to the average idle time per station.

Arcus (1966) constructed an approach which uses weighted sampling for selection of the task to be next assigned from the list of available and feasible tasks. He weighted each task on five different parameters. The composite weight associated with the task is the product of these five weights. A line is balanced several times and the best assignment is retained. Arcus also proposed a mechanism for fathoming a solution prior to its completion if it is dominated by some other solution with respect to the total idle time. Tonge's (1965) method assigns tasks to workstations by randomly selecting the heuristic to choose the next task to be assigned to the current workstation. A line is balanced many times using different combination of heuristics and the best one is selected. Tonge concludes that random selection of heuristics for selecting the task to be assigned next is superior to random choice of tasks directly or the use of individual heuristics alone.

In a recent evaluation of these line balancing techniques, Talbot, Gehrlein and Patterson (1985) concluded that the variants of Hoffman's procedure yielded the best results among the heuristic approaches, while among optimum-seeking methods Wee and Magazine's procedure which used the adaptation of the first fit decreasing rule, followed by Talbot and Patterson's procedure was the most efficient.

6.2 Mixed Model Problems

In mixed model assembly lines, different jobs with significantly different work content are processed simultaneously. Most of the research done in this area is based on task assignment to operators in assembly lines where the the cycle time at each workstation is determined by the speed of the conveyor and the length of the workstation.

The mixed model ALB problem was first identified by Kilbridge and Wester who presented the variable rate and the fixed rate parts input strategies.

Dar-El (1978) categorized mixed model ALB problems in terms of the launching discipline, the characteristics of the workstations, the conveyance system and the manner in which the products are linked to the conveyance system. Thomopolous (1967) considered the problem of balancing the workload assigned to the operators. His procedure is based on Kilbridge and Wester's method, modified to consider the production requirements for the entire shift. In his method, balance of workloads among different operators is achieved on a shift basis rather than on a cycle time basis. Sequencing is based on selecting next the model which would incur the minimum cost of inefficiency and whose demand has not been met so far. Thomopolous (1970) considered the problem of smoothing the workstation loads for each model. He generates a composite precedence relationship for all model types and proceeds to minimize the total absolute deviation of the workload for any model at any one workstation from the average workload through a serial procedure.

MacAskill (1972) divided the mixed model problem into two subproblems: i) allocation of total work content to the workstations, and ii) launching and sequencing of these models. His paper, however, addresses only the first subproblem. In his approach, aggregate task groups are formed, based on the combined precedence relationship for all the model types. He used Helgeson and Birnie's heuristic method for selecting the next task for assignment.

Dar-El and Cother (1975) considered the problem of minimizing the station lengths

for the case where the jobs are fixed to the conveyor which is moving at a constant speed. They presented heuristic algorithms for both open and closed stations. Dar-El and Cucuy (1977) utilized this approach for balancing the workload for each model across all the workstations.

7 FLEXIBLE MANUFACTURING SYSTEMS

A flexible manufacturing system comprises a set of NC or CNC machine tools, linked by an automated material handling system. Any efficient approach towards the scheduling of such systems must take account of the versatility of machines, existence of alternative routings, travel times of material handling equipment, consideration of storage space and the like.

Several papers on flexible manufacturing systems have attempted to provide a conceptual framework for analyzing the related management problems; notable among these being the articles by Chen and Talavage (1982), Stecke (1984), Eversheim and Hermann (1982) and Suri and Whitney (1984). The papers by Buzacott and Yao (1982), Stecke (1985) and Suri (1984) provided excellent discussions on the mathematical modeling approaches for analyzing these issues. We will now discuss these management issues as background for the scheduling problem.

7.1 Design Problems

Design issues relate to the specification of desired flexibility, selection of part types to be manufactured, design and lay-out of the machine tools, material handling equipment and transport systems and design of support systems. Browne, Chan and Rathmill(1984a) discussed these issues in some detail.

Groover (1980) classified flexible manufacturing systems into *dedicated* and *random*

systems. A dedicated FMS produces a fixed set of parts with relatively stationary demand requirements over a specified time horizon. In contrast, a random FMS manufactures a larger variety of parts with arbitrary demand patterns. The flexibility requirements for these two systems would clearly be different. Browne, Dubois, Rathmill, Sethi and Stecke (1984b) classified flexibility requirements along eight dimensions of machine flexibility, process flexibility, product flexibility, routing flexibility, volume flexibility, expansion flexibility, operation flexibility and production flexibility. They also categorized FMS's into flexible manufacturing cells, flexible machining systems, flexible transfer lines and flexible transfer multi-lines, depending upon the flow of materials and the complexity of operations. Chatterjee, Cohen, Maxwell and Miller (1984) attempted to quantify flexibility. They decomposed an FMS into module centers – a set of machine tools with a dedicated transport system. The network of module centers and the transport system can then be suitably quantified to develop measures of product, routing and process flexibility.

Clearly, the extent of flexibility attainable in practice depends upon the type of FMS used as well as the characteristics of the material handling system. Stecke and Brown (1984) studied the combination of four types of FMSs with six different types of material handling systems. The resulting flexibility was ranked along the eight dimensions indicated in Browne, Dubois, Rathmill, Sethi and Stecke (1983b). Based on a study of thirty major FMS installations, they concluded that the inherent flexibility of many systems is not being exploited fully. Dupont-Gatelmand's (1982) survey of the existing flexible manufacturing systems provided a fairly exhaustive coverage of the machine tool and material handling equipment in use. This survey also indicates a wide variation in the degree of flexibility achieved in these systems.

Whitney and Suri (1984) formulated the selection of machines and the parts to be manufactured in the FMS as an integer programming problem; two heuristic procedures have been proposed for solving these problems. The first procedure, PAMS, essentially determines the number of machines required and provides a first cut approximation for the

selection of parts. The second heuristic procedure, PARSE, uses a sequential decision algorithm, at a more detailed level, to determine the 'optimum' set of parts.

Several authors have studied the impact of the size of the buffers on the performance of a FMS. Dubois's (1983) study, based on a semi-open queueing network representation of the system, demonstrated the asymptotic behavior of the production rate with respect to the level of inventory permitted. Simulation studies also indicate that analytical models tend to give pessimistic estimates for machine utilizations and queue lengths. Gershwin (1982) looked at the problem of optimizing buffer sizes from the standpoint of reducing the impact of machine breakdowns. In-process inventory tends to reduce the effect of these breakdowns by isolating the machines. While optimum buffer sizes can be determined analytically for two- and three-stage systems, there is an exponential growth in the problem size with increase in state space. For larger problems, Gershwin proposed deriving bounds either by underestimating the contribution of one or more machines and associated buffers, or by aggregating individual machines into groups. Buzacott and Shantikumar (1980) showed the superiority of a common storage over local storages at individual machines.

Maxwell and Muckstadt (1982) addressed the problem of determining the optimum number of automated guided vehicles for an assembly system. In a two-step procedure, they first derive the minimum number of vehicles required, disregarding track and space congestion and time-phased material handling requirements, by solving a transportation problem for minimizing the number of idle vehicle movements. In the second step, they generate the number of routes and trips required for the shift. Assignment of vehicles to trips is done through a heuristic dispatching procedure, conflicts being resolved by giving higher priority to shorter trips. Solution improvement is sought through a pairwise interchange procedure. The authors used simulation studies to study blocking effects and to evaluate alternative assignments. Egbelu and Tanchoco (1984) proposed and tested several heuristic dispatching rules for automated guided vehicles. These rules have been

categorized into the those which are initiated by the workstations when it has more than one vehicle to choose from, and those which are initiated by the vehicle when several workstations are competing for service. Simulation studies indicate that for large volumes of material flow, the vehicle initiated rules are dominant, though some of these rules could lead to system deadlock.

7.2 Planning Problems

Planning problems can be decomposed into batching problems – selection of a batch of parts for concurrent manufacture, and loading problems – assignment of operations to machines. Stecke (1983a) additionally considered the machine grouping problem. The objective sought has been almost uniformly to maximize the production rate, or equivalently, to minimize batch completion time.

The bulk of the research aimed at maximizing production rate has been based on a queueing network representation of the flexible manufacturing system. Initial work in this area was done by Solberg (1977), who modeled a FMS as a closed network of queues and derived a product form solution for the production rate under the assumptions of first come first served queue discipline, exponential service times and no equipment failures. His CAN-Q procedure can be used, albeit at an aggregate level, to derive the interrelationship among the production rate, machine utilization, and a number of other system parameters. Solberg found the results to be surprisingly robust in spite of the limiting assumptions made in formulating the model. Suri and Hildebrandt's (1980) MVAQ program, based on the mean value analysis of a closed network of queues, permits the modeling of multiple part-classes and, compared to CAN-Q, can cover greater detail with less memory requirements. Simulation studies have indicated that MVAQ can be accurate within 10%–15%. Buzacott and Shantikumar (1980) used an open queueing network model under the assumptions of exponential interarrival and processing times. For a given number of jobs

in the queue, production rates have been derived for a variety of production systems for different input strategies and buffer capacities. The authors also derived expressions for the expected number of parts in the system and effective production rates after considering equipment breakdowns. Seidman, Schweitzer and Nof (1984) studied a unitary manufacturing cell – a group of flexible, automated workstations with stochastic processing times, connected by a material handling robot. The rejected parts are rerouted for rework. For the case of single part type and arbitrary distributions of the processing and inspection times, they derived the distribution for total batch processing time. Expressions for the probability density functions of the time elapsed between successive production completions, within each part type and over several part types, have also been derived. Their results indicate that simultaneous production of several part types does not improve the productivity achieved with batch production of these part types individually.

Not much work has ben done on solving the batching problem so far. Some of the researchers have introduced concepts of Group Technology in forming part families. However, as Nof, Barash and Solberg (1979) have emphasized, the design and operational objectives in selecting parts for concurrent manufacture are at variance with one another. While at the design stage, it is important to identify similarities among different parts, operational objectives of maximizing system throughput and machine utilization favor part types which are complementary in their processing requirements. Chakravarty and Shtub (1984) avoided this conflict by forming clusters of parts based on tool requirements. The batching procedure suggested by Whitney and Gaul (1984) is based on sequential decision process. In their list processing algorithm, each part is ranked in descending order of its 'probability of successful outcome'. This index favors parts which have low slack, which can be fabricated with versatile tools and which tend to balance machine utilization. The authors proposed a similar approach for the balancing problem as well.

While the objective of balancing workloads on individual machines is intuitively appealing, as it reduces the load on the bottleneck machine, Stecke and Solberg's (1984) study indicated the efficacy of unbalancing. Using a closed queueing network representation of the system, they have shown that production rate is maximized by 'pooling' machines and assigning heavier workload per machine to the larger group. While the optimum workloads can be derived analytically for small systems, searching over all possible workload parameters is required for larger problems. Machine pooling also increases flexibility by increasing the number of available job routings and improves system robustness against equipment breakdowns. The improvements obtained with pooling decreases with an increase in the number of parts in the system and in the limiting case, both balanced and unbalanced workloads yield the same rate of production. This confirms the conclusions reached earlier by Buzacott and Shantikumar (1980) who demonstrated the optimality of balanced workloads for systems with infinite storage capacity. Stecke (1981) indicated that balancing is appropriate also for deterministic systems with fixed routes, in systems with no pooling of machines or with groups of equal sizes. It may be optimal for some other systems as well because of the discreteness of processing times. Stecke and Morin (1985) showed the optimality of balancing for systems with fixed routes – each operation being assigned to only one machine, and infinite buffer space. Using a closed queueing network model, they established the production function to be quasiconcave and symmetric over the range of feasible workloads. The optimality of the symmetric point, which represents balanced workloads, follows consequently. Stecke and Shantikumar (1984) showed that balanced workloads tend to minimize in-process inventory stochastically. This result holds for three different input release strategies.

Stecke (1983b) proposed a hierarchical approach for solving the grouping and loading problems. At an aggregate level, the grouping problem is solved by pooling machines as much as possible to meet technological restrictions. Solution of the aggregate loading problem requires assigning specific unbalanced workloads to groups of unequal

sizes. Groups with the same number of machines are assigned equal workloads. At a detailed level, the grouping problem is formulated as a nonlinear mixed integer programming problem to minimize the number of groups required. Machines are then assigned to these groups based on the aggregate grouping solution. Stecke (1983a) proposed several objectives of the detailed loading problem. These include, i) balancing workloads over all the machines or machine groups, ii) achieving a specific unbalance in the workloads assigned to different groups, iii) minimizing movement between machines, and iv) filling the tool magazine as densely as possible, to maximize process flexibility. For each of these objectives, the loading problem is formulated as a nonlinear mixed integer programing problem. Alternative methods for linearizing the product terms have been presented and compared for their computational requirements. Stecke and Talbot (1983) have proposed heuristic loading algorithms for the objectives of i) minimizing the movement of parts between machines, ii) assigning balanced workloads to groups of equal sizes, and iii) maximizing workload unbalance when the groups have unequal sizes.

Stecke and Berrada (1984) proposed a branch and bound algorithm for balancing workloads by minimizing the workload on the bottleneck machine. Their procedure requires generation of subproblems to establish new upper and lower bounds on the solution values. These bounds are used to revise the maximum permissible workload for the subsequent subproblem. The procedure is terminated when the upper and the lower bounds are 'acceptably' close. The subproblems are solved by a branch and bound method. the nodes representing a partial assignment of operations. At each of these nodes, a relaxed problem is obtained by disregarding the workload and tool magazine constraints. A binary knapsack problem is subsequently solved to revise the optimal solution to account for the violated constraints. The penalty associated with the reassignment of any operation is the marginal increase in its processing time. The branching rule favors a reassignment to machines with lower workloads and larger values of available tool space.

Kusiak (1984a) formulated the loading problem as generalized assignment and

generalized transportation problems. These formulations have been further extended to account for constraints associated with the length of remaining tool life and the maximum allowable processing time for a batch of identical parts at any workstation. In another paper, Kusiak (1984b) developed models for the optimal tool path – the optimal sequence of operations for any part, based on a set partitioning approach. The problem of sequencing these tool paths is shown to be similar to the single machine scheduling problem with precedence constraints. If the cost of these tool paths is sequence-dependent, the resulting problem is similar to the traveling salesman problem. The paper, however, does not bring out clearly how these costs are derived. Chakravarty and Shtub's (1984) procedure first assigns parts to tools; the subsequent assignment of tools to machines is formulated as a 0–1 integer programming problem. Ammons, Lofgren and McGinnis (1984) proposed heuristic procedures for the objectives of minimizing workload imbalance and minimizing the movement of parts. A third heuristic method has been proposed for achieving a compromise between these two conflicting objectives.

Jaikumar, Maxwell and Stecke (1985) developed an integer programming formulation for the loading problem with the objective of balancing workloads. The Lagrangean problem, obtained by relaxing the assignment and tool magazine capacity constraints, is decomposed into subproblems for fixed values of the multipliers. The subproblem, which has the structure of a generalized assignment problem, is solved by a multiplier adjustment method. The multipliers for the original Lagrangean problem are updated by using a subgradient optimization procedure.

7.3 Scheduling and Control Problems

Buzacott (1982) compared the relative efficiencies of determining the sequence of operations at the pre-release, input and operational levels. He concludes that when alternative routings exist, the production capacity of the system can be increased

significantly if the sequencing decisions are made at the input and the operational levels. His study also indicates that further improvement is possible when the input control level incorporates information about the system status at the time of releasing the next job.

Morton and Smunt's (1984) hierarchical approach consists of making strategic and capacity planning decisions prior to scheduling. The objective of the scheduling problem is to minimize the opportunity cost of machine usage, which is determined by the solution to the capacity planning problem, and the penalty costs associated with job tardiness. Jobs are assigned to machines which yield minimum marginal cost. The dispatching rule for selecting the next operation on any machine is based on a priority index which favors jobs with high marginal cost of tardiness and which are most likely to be tardy if the imminent operation is delayed.

Hildebrandt (1980) proposed a hierarchical approach to the scheduling problem which incorporates machine failures. The batch completion time, which needs to be minimized, is decomposed into time-phases corresponding to individual failure states of the system. The first level determines the part-mix and part-routings by solving a nonlinear programming problem, the nonlinearity arising from the constraints on the total number of pallets/ fixtures available for a given part and on the maximum number of parts permitted in the system at any given time. Additionally, this formulation requires the average time spent, in waiting and for processing, that a part spends at any machine. Hidebrandt uses the mean value analysis approach to estimate this parameter. The nonlinear problem is solved by using a successive linearization approach in which, starting with a feasible value, the batch completion time is decreased by small amounts and the change in the pallet/fixture requirements to satisfy all constraints is recorded. The procedure is terminated when it is no longer feasible to reduce the batch completion time. Given the parts and their routes, the second level determines the optimal input sequence for these parts. To minimize the waiting time at each station, Hildebrandt proposes a scheme by which the machines are

provided an even flow of work. He constructs a recursion relation, which minimizes the absolute deviation of actual workload from the average workload for the system, to develop a dynamic programming algorithm. This scheme has been shown to be superior to the SPT rule for minimizing the total batch completion time. At the third level, decisions regarding the selection of the part to be processed next, from among those waiting for the machine, have to be made. Hildebrandt proposed two heuristic dispatching rules for solving this problem.

The hierarchical approach of Kimemia (1982), Kimemia and Gershwin (1983), Gershwin, Akella and Choong (1984) and Akella, Choong and Gershwin (1984) for solving the scheduling problem for an unreliable system is based on formulating a flow control problem to determine the optimum production rates in any failure state of the system at the top level. The objective function is based on the current system status and the costs associated with the earliness and lateness for each part. For a given state, the optimal policy divides the buffer state space into a number of regions, the optimal production rate remaining a constant within each region. The 'hedging' point, defined for each failure state for which the demand rate is feasible, determines the optimal buffer levels for each part after considering possible future machine failures. The complex flow control problem has been simplified by decomposing it into two simpler subproblems; the objective function values of these subproblems define the upper and the lower bounds on the objective value of the original problem. For a given state, efficient production rates can be determined by solving a linear program whose objective function coefficients are derivatives of the optimal solution function of the flow control problem. At the lower levels, the routes and the input sequence for each part are determined to ensure the production rates specified at the top level.

Stecke and Solberg (1981) studied the interaction between loading policies and dispatching rules in a simulation study. They found the SPT/TOT dispatching rule to be superior for all loading policies. The loading objectives of minimizing part movements and

assigning unbalanced workload to totally pooled machines yielded the maximum production rate.

The FMS scheduling problem has most of the complexities of a conventional multi-stage scheduling problem. In an FMS however, the number of these stages need not be predetermined and each stage may have a number of parallel machines. Additional complexities are introduced because of the considerations of possible equipment failures and interactions of the processing times with the travel times of the material handling system. While some progress has been made in analyzing these issues, there is ample scope for future research.

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Appendix 1

A Discrete Plant Location Problem

A simple discrete plant location problem can be formulated as below:

$$\text{Minimize } Z = \sum_{i=1}^{m} f_i Y_i + \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} X_{ij}$$

subject to:

$$\sum_{i=1}^{m} X_{ij} = d_j \qquad j=1,\ldots,n \qquad (1)$$

$$\sum_{j=1}^{n} X_{ij} \leq Y_i \sum_{j=1}^{n} d_j \qquad i=1,\ldots,m \qquad (2)$$

$$X_{ij} \geq 0 \qquad \text{for all } i, j \qquad (3)$$

$$Y_i \in (0,1) \qquad \text{for all } i \qquad (4)$$

This formulation specifies m potential sites for locating the facility(ies), to serve n customers. The fixed cost for locating facility at site i is f_i ; the transportation cost for supplying one unit of finished goods from site i to customer j is c_{ij}.

Constraint (1) specifies that the demands from all the customers should be satisfied fully. Constraint (2) ensures that material flow will take place only from sites which are selected. Constraints (3) and (4) indicate the nature of the variables.

Appendix 2

A Model for Aggregate Planning

A fixed cost model for the aggregate planning problem is given below:

$$\text{Minimize } Z = \sum_{i=1}^{N}\sum_{t=1}^{T}(s_{it}\Delta(X_{it}) + v_{it}X_{it} + c_{it}I_{it}) + \sum_{t=1}^{T}(r_tW_t + o_tO_t + h_tH_t + f_tF_t)$$

subject to:

$$X_{it} + I_{i,t-1} - I_{it} = d_{it} \qquad i=1,\ldots,N; \quad t=1,\ldots,T \qquad (1)$$

$$\sum_{i=1}^{N} a_i\Delta(X_{it}) + k_tX_{it} - W_t - O_t \le 0 \qquad t=1,\ldots,T \qquad (2)$$

$$W_t - W_{t-1} - H_t + F_t = 0 \qquad t=1,\ldots,T \qquad (3)$$

$$-pW_t + O_t \le 0 \qquad t=1,\ldots,T \qquad (4)$$

$$X_{it}, I_{it} \ge 0 \qquad i=1,\ldots,N; \quad t=1,\ldots,T \qquad (5)$$

$$W_t, O_t, H_t, F_t \ge 0 \qquad t=1,\ldots,T \qquad (6)$$

$$\Delta(X_{it}) = 1 \text{ if } X_{it} \ge 0; \quad = 0 \text{ otherwise.} \qquad i=1,\ldots,N; \quad t=1,\ldots,T \qquad (7)$$

X_{it} denotes the production quantity for product i in period t; the setup cost is denoted by s_{it}. v_{it} denotes the unit variable production cost for product i in period t. c_{it} is the unit inventory carrying cost per period. r_t and o_t are the unit regular time cost and the unit overtime cost, respectively. h_t and f_t are, respectively, the cost of hiring and and the cost of laying off one employee. I_{it}, W_t, O_t, H_t, F_t denote the inventory of product i at the end

of period t, the size of the work force at the end of period t, the amount of overtime during period t, the number of employees hired during period t, and the number of employees laid off during period t, respectively. k_i is the unit processing time for product i. p is the ratio of the maximum overtime permitted to the size of the work force.

The model attempts to minimize the total cost over the planning horizon T. Constraint (1) ensures the material balance between successive periods. Constraint (2) indicates the capacity restrictions. Constraint (3) ensures the balance in the size of of the work force. Constraint (4) represents the limitation on the total overtime in any period. Other constraints indicate the nature of the variables.

Appendix 3

List of Notations

Symbol	Particulars
J_i	Job i, i=1,2,....n
M_j	Machine j, j=1,2,...m
P_{ij}	Processing time for job i on machine j
d_i	Due date for job i
C_i	Completion time of job i
L_i	Lateness of job i, $L_i = C_i - d_i$
T_i	Tardiness of job i, $T_i = \max(0, L_i)$
U_i	Tardiness index of job i, $U_i = 1$, if $T_i > 0$ $U_i = 0$, otherwise
w_i	Priority weight for job i
q_i	$\max_i\{d_i\} - C_i$,
r_i	Ready time for job i
$\bar{F}$	Mean unweighted flow time
$\bar{F}_w$	Mean weighted flow time
$\bar{C}$	Mean unweighted completion time

Symbol	Particulars
$\dot{C}_w$	Mean weighted completion time
$\dot{T}$	Mean unweighted tardiness
$\dot{T}_w$	Mean weighted tardiness
$\dot{U}$	Mean unweighted number of tardy jobs
$\dot{U}_w$	Mean weighted number of tardy jobs
I	Shop with identical parallel machines
U	Shop with uniform parallel machines
R	Shop with unrelated parallel machines
F	Flow shop
P	Permutation flow shop
J	Job shop
SPT	Shortest processing time
WSPT	Weighted shortest processing time
LPT	Longest processing time
S/OPN	Slack per operation
FT/OPN	Flow time per operation
TWK	Total work content
EDD	Earliest due date
MDD	Modified due date
WINQ	Work in next queue

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11. ABSTRACT (A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here)

This paper presents a survey of the existing literature on machine scheduling from the perspective flexible manufacturing systems. It is the first of a series of papers planned to document research in scheduling for the Automated Manufacturing Research Facility (AMRF) at the National Bureau of Standards.

An overview of the hierarchical production planning process is given. Scheduling appears at the lowest level of this hierarchy where current information is available and execution is carried out repetitively within relatively short time horizons.

The scheduling problem is also classified along the dimensions of order characteristics, processing environment, and scheduling objective. The paper covers deterministic, non-preemptive scheduling of open shops, since a typical flexible manufacturing system (FMS) operates under these conditions. Both due-date based and flow-time based objectives are addressed for single stage single machine, single stage parallel machines, flow shops and job shops. Research in assembly line balancing is similarly covered since it is possible to treat an FMS as a transfer line for repetitive discrete manufacture.

The FMS-related management problems are classified into, i) Design problems, ii) Planning problems, and iii) Scheduling and Control problems. The analytical approaches to these problems have focussed primarily on the objectives of maximizing production rate, minimizing in-process inventory, and maximizing machine utilization.

12. KEY WORDS (Six to twelve entries; alphabetical order; capitalize only proper names; and separate key words by semicolons)

Automated manufacturing; real-time scheduling; robotics; job shop; flow shop

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